

Math Aptitude Test #1

Notes:

1. This exam consists of two parts, each part contains 10 questions. You have to show your answers properly. Partial credits are considered for steps.
2. Scientific Calculators are not allowed, only basic ones.

Part (I): consists of ten questions – show your logical steps

1. Simplify $\frac{x^{-1} + x}{(1 - x^{-1})^{-1}} =$

a. $\frac{2x}{1-x}$

b. $\frac{x^2 + 1}{x(1-x)}$

c. $\frac{(x^2 + 1)(x-1)}{x^2}$

d. $\frac{(1-x+x^2-x^3)}{x}$

2. Simplify $(x+1)(x^2 + 3x - 5) - 3x^2 =$

a. $x^3 + 4x^2 - 2x - 5$

b. $x^3 + x^2 - 2x - 5$

c. $-2x^3 + 4x^2 - 2x - 5$

d. $-2x^2 + 4x - 4$

3. Use synthetic division to express $p(x) = 7x^3 - 14x^2 - 18x - 20$, in the form of $(\text{divisor})(\text{quotient}) + \text{remainder}$ for the divisor $(x-3)$.

a. $(x-7)(11x^2 + 7x + 3) - 11$

b. $(x-3)(7x^2 + 7x - 3) - 11$

c. $(x-3)(7x^2 + 3) - 11$

d. $(x-3)(7x^2 + 7x + 3) - 11$

4. The solution of the system $\begin{cases} \frac{2}{3}x - \frac{1}{2}y = \frac{5}{6} \\ \frac{2}{5}x - \frac{3}{10}y = \frac{1}{2} \end{cases}$ is

a. $x = 5, y = -1$

b. $x = \frac{4}{3}, y = \frac{3}{5}$

c. No solution

d. Infinitely many solutions

5. The domain of $f(x) = \frac{1}{x^2 - 16} \sqrt{x+2}$ is

a. $[2, \infty)$

b. $[-2, \infty)$

c. $[-2, \infty) \setminus \{4\}$

d. $[-2, \infty) \setminus \{-4, 4\}$

6. Let x, y, z be positive numbers and $b \in (0, \infty) \setminus \{1\}$. Use the properties of logarithms to simplify

$$\log_b \sqrt[4]{\frac{x^7 y^2}{z^4}} =$$

a. $\frac{7}{4} \log_b(x) + \log_b(y) - \log_b(z)$

b. $\frac{7}{4} \log_b(x) + \frac{1}{2} \log_b(y) - \log_b(z)$

c. $\frac{7}{4} \log_b(x + y - z)$

d. $\frac{7}{4} \log_b(x) - \log_b(y) - \log_b(z)$

7. The equation $\sqrt{2x} + \sqrt{x-4} = 2$ has exactly how many roots

a. None

b. One

c. Two

d. Three

8. One factor of $2x^2y^3 - 4xy^4$ is

a. $4x^2y^4$

b. $2x^2y^3$

c. $2xy^4$

d. $x - 2y$

9. Simplify $\frac{x^2 - x - 6}{x^2 + 8x + 16} \div \frac{x^2 - 9}{x^2 - x - 20} =$

a. $\frac{(x+2)(x-3)}{(x-5)(x+4)}$

b. $\frac{(x-2)(x+5)}{(x-3)(x+4)}$

c. $\frac{(x+2)(x-5)}{(x+3)(x+4)}$

d. $\frac{(x-2)(x-5)}{(x-3)(x+4)}$

10. The roots of $\sqrt{13+x} - x = 7$ are

a. -4 and -9

b. -4 and 9

c. Only -9

d. Only -4

Part (II): consists of ten questions – show your logical steps

1. Let $\vec{A} = 4i + j - 3k$ and $\vec{B} = -3i + 5j + k$, then their cross product is

a. $16i + 5j + 23k$

b. $16i - 5j + 23k$

c. $-12i + 5j - 3k$

d. $16i - 5j + 23k$

2. The determinant of $\begin{vmatrix} -2 & 5 \\ -3 & -3 \end{vmatrix}$ is

a. 21

b. 9

c. 6

d. -9

3. If $u = \left\langle -\frac{1}{3}, -\frac{3}{2} \right\rangle$ and $v = \langle 2, 9 \rangle$ are two vectors. Determine whether they are parallel, orthogonal or neither

a. Parallel

b. Orthogonal

c. Neither

4. The solution set of the inequality $\frac{x^2 - 1}{x^2 - 5x + 6} > 0$ is
- $(1, 2), (3, \infty)$
 - $(-1, 1), (2, 3)$
 - $(-\infty, -1), (1, 3)$
 - $(-\infty, -1), (1, 2), (3, \infty)$
5. If $y = u^5 - 8u^2 + 2u - 1$ and $u = \sqrt{x + 10}$, then $\frac{dy}{dx}(x = -9)$ is
- 0
 - 1
 - 1
 - 9
6. If $xe^x + y(\ln x) + y^2 = 3$, then by using implicit differentiation $\frac{dy}{dx}$ at $p(1, e)$
- 0
 - $3/2$
 - $-3/2$
 - $-2/3$
7. The value of $\lim_{x \rightarrow 3} (x\sqrt{x^2 - 9}) =$
- 0
 - $-\infty$
 - $+\infty$
 - Does not exist
8. Use integration by parts to find $\int_0^3 \left(\frac{x}{3}\right) e^{x/3} dx =$
- 0
 - 3
 - 6
 - 9
9. Let $y = \frac{x^2}{x-1}$, define $\forall x \in \mathbb{R} \setminus \{1\}$. Then $\frac{dy}{dx} =$
- $\frac{x(3x-2)}{(x-1)^2}$
 - $\frac{(x^2+1)}{(x-1)^2}$
 - $\frac{x(x-2)}{(x-1)^2}$
 - None of the above

10. The value of $\int \frac{x^2 + 4x - 3}{x - 1} dx$

a. $\frac{1}{2} \ln|x-1| + C$

b. $\frac{1}{3} \ln|x-1| + C$

c. $\frac{x^2}{2} + 5x + 2 \ln|x-1| + C$

d. $\frac{x^2}{2} + 6x + 3 \ln|x-1| + C$

Good Luck